# LEO Satellite Networks: When Do All Shortest Distance Paths Belong to Minimum Hop Path Set?

Quan Chen, Lei Yang, Deke Guo, Bangbang Ren, Jianming Guo, and Xiaoqian Chen

Abstract—Low-Earth orbit (LEO) satellite constellation network (SCN) has become a promising solution for non-terrestrial networks (NTNs). In LEO-SCNs, the shortest distance path (SDP) and minimum hop path (MHP) are two types of important paths.This paper focuses on the proposition that all the SDPs belong to the MHP set and studies the conditions when the proposition holds or not. Based on the topological regularity and link distance variation patterns, this paper proves several simplified equivalent propositions and derives a discriminant function to judge if the proposition holds in an arbitrary constellation. Simulations verify the judging method and find that all the SDPs belong to the MHP set in constellations with small inclinations (less than 68 deg) or large phasing offsets. The propositions can help to simplify the calculation of SDP.

Index Terms—LEO satellite networks, constellation, routing, inter-satellite link, shortest path, minimum hop path

## I. INTRODUCTION

Low-Earth orbit (LEO) satellite constellation networks (SCNs), especially mega-constellation networks, have become an emerging technology for providing low-latency, broadband, and wide-area network services [1], [2]. Studies have shown that the LEO-SCNs have low-latency advantages over ground networks in many scenarios [3].

The delay in LEO-SCN is mainly determined by the path distance, thus many routing strategies search for the *shortest distance paths* (SDPs) [4] which can be solved by classical shortest path algorithms, e.g., Dijkstra algorithm [5]. The minimum hop-count path (MHP) is also an important type of path that has the least possible hops between two nodes [6]. In general wireless networks, it is difficult to find an explicit relation between SDP and MHP if the links have irregular or random lengths. Between two nodes, an SDP may have more hops than the MHP. However, in LEO-SCN, it is possible that all SDPs belong to the MHP set because of the regularity and symmetry of the constellation topology and the periodic variation of inter-satellite links (ISLs) [4]. If this proposition holds, then the SDP calculation in the constellation can be greatly simplified and limited to a smaller sub-graph.

Generally, the LEO-SCN has a mesh-like topology [3], and thus the calculation of MHP in LEO-SCN is much simpler than SDP. Moreover, since those MHPs define a local subgraph, if SDP is strictly proved to belong to the MHP set, the solution of SDP can be simplified to within the subgraph, which greatly lowers the complexity especially in mega-constellations.

1

Qu et al. [5] summarize the SDP and MHP and their applications in routing strategies. Chen et al. [7] assume all SDPs are also MHP and adopt the MHP instead of SDP as the routing metric for low-latency. Also, some researchers have investigated if all SDPs belong to the MHP set. Ekici et al. [6] find the SDP is in the MHP set in some special cases, then they exploit the latitude information to simplify the shortest path calculation. But this algorithm only applies to polar constellations with zero phasing offset. Duan [8] proposes a judging condition for all SDPs belong to the MHP set in polar constellation networks. But the condition is sufficient but not necessary and only applies to limited cases. Therefore, a general and analytical judging approach is needed.

This paper aims to provide a general method for judging if all SDPs belong to the MHP set in a given LEO-SCN. The original proposition is equivalently converted and simplified. Then an explicit discriminant function is derived to judge if the proposition holds or not. The judging condition is sufficient and necessary. Finally, numerical simulations verify the method. The main contributions are summarized as follows:

- In the typical constellation network topology, judging if all SDPs belong to the MHP set is equivalently simplified to judge the single-hop vertical detour case.
- Based on the ISL variation, a discriminant function and analytical criterion are proposed to judge if all SDPs in a given constellation network belong to the MHP set or not.
- Simulations verify the proposed judging method and the results show that whether the proposition holds mainly depends on the orbit inclination and phasing factor. All SDPs belong to the MHP set in constellations with small inclinations (less than 68 deg) or large phasing offsets.

#### II. CONSTELLATION NETWORK MODEL

## A. Satellite Constellation and ISLs

The LEO-SCN typically adopts the Walker constellation [5] that is formally expressed by  $\alpha$ :  $N_S/N_P/F$ , where  $\alpha$  is the orbit inclination,  $\alpha \in (0, \pi)$ ,  $N_S$  is the total satellite number,  $N_P$  is the number of orbit planes, and F is a phasing factor.  $N_P$  orbit planes with the same inclination and altitude are regularly distributed along the equator, and these orbits are evenly spaced by  $\Delta\Omega = 2\pi/N_P$ .  $M_P$  satellites are also evenly distributed in each plane,  $M_P = N_S/N_P$ . The satellite position within the plane is specified by the satellite phase angle u ( $u \in [-\pi, \pi]$ ). The phase angle differences of adjacent

© 2022 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See https://www.ieee.org/publications/rights/index.html for more information.

Q. Chen, L. Yang and J. Guo are with the College of Aerospace Science and Engineering, National University of Defense Technology, Changsha, China. D. Guo and B. Ren are with the Science and Technology Laboratory on

Information Systems Engineering, National University of Defense Technology.

X. Chen is with the National Innovation Institute of Defense Technology, Chinese Academy of Military Science, Beijing, China.



Fig. 1. Constellation topology and ISLs. All V-ISLs have equal and constant distances, while H-ISL distances are different and vary with satellite phase u.

satellites within the plane and between adjacent planes are  $\Delta \Phi = 2\pi/M_P$  and  $\Delta f = 2\pi F/N_S$ , respectively.

Fig. 1 shows the classical ISL connecting mode [4]. Each satellite maintains four ISLs: two vertical (intra-plane) ISLs (V-ISLs) with adjacent satellites within plane; two horizontal (inter-plane) ISLs (H-ISLs) with satellites in adjacent orbits.

## B. ISL distance variation

Because of the regularity and symmetry of the Walker constellation, the ISL variation pattern of any satellite applies to all the satellites in the constellation. The ISL distance is

$$d = \sqrt{2} \left( R_E + h_s \right) \sqrt{1 - \cos \Theta} \tag{1}$$

where  $R_E$  is the Earth radius,  $h_s$  is the orbit altitude, and  $\Theta$  is the Earth-centered angle between the two satellites.  $\Theta$  of V-ISLs is  $\Delta \Phi$ , while  $\Theta$  of H-ISLs can be calculated by

$$\cos\Theta = c_1 - c_2\cos(2u + \Delta f) \tag{2}$$

where  $c_1 = (\cos^2(\Delta\Omega/2) - \cos^2\alpha \sin^2(\Delta\Omega/2)) \cos \Delta f - \cos \alpha \sin \Delta\Omega \sin \Delta f$ ,  $c_2 = \sin^2\alpha \sin^2(\Delta\Omega/2)$ . Based on (1) and (2), the ISL distances have two characteristics:

- All the V-ISLs are of the same and constant distances;
- H-ISL distances are different and vary with satellite phase *u*. In terms of H-ISLs, the derivative of *d* is

$$\frac{\mathrm{d}d}{\mathrm{d}u} = -\frac{\sqrt{2}\left(R_E + h_s\right)}{\sqrt{1 - \cos\Theta}}c_2\sin(2u + \Delta f) \tag{3}$$

Since  $c_2 > 0$ ,  $\sqrt{1 - \cos \Theta} > 0$ , the sign of  $\frac{dd}{du}$  is determined by  $\sin(2u + \Delta f)$ . The H-ISL distance reaches the minimum at  $u' = \frac{2k+1}{2}\pi - \frac{\Delta f}{2}$  and the maximum at  $u'' = k\pi - \frac{\Delta f}{2}$ . The distance variation has a period of  $\pi$  and is symmetrical about  $u = \frac{k\pi}{2} - \frac{\Delta f}{2}$ , as later shown in Fig.6.

## III. NOTATIONS AND DEFINITIONS

With the four-ISL connecting pattern, the virtual topology of the constellation network is mesh-like or torus-like [3], [9] (see Fig. 2). Each satellite S can be identified by a virtual address  $(v_S, h_S)$  indicating the  $v_S$ -th satellite in the  $h_S$ -th orbit plane. Any two satellites in the network are reachable via a multihop path P. The vertical, horizontal, and total hop-counts of P are denoted by  $N^v(P)$ ,  $N^h(P)$ , and N(P), respectively. The physical distance of P is d(P).



2

Fig. 2. The mesh-like virtual topology of constellation. The SDP calculation can be simplified to the MHP region if all SDPs belong to MHP set.



Fig. 3. Examples of the horizontal detour. The dashed line indicates the corresponding minimum hop path.



Fig. 4. Examples of the vertical detour. The dashed line indicates the corresponding minimum hop path.

**Definition 1.** *Minimum Hop Path (MHP).* In the meshlike topology, the path from satellite  $A(v_A, h_A)$  to  $B(v_B, h_B)$ ,  $P_{AB}$ , is not unique. The MHP between A and B is defined as  $P_{AB}^0 = \arg \min_{P \in \{P_{AB}\}} N(P)$ . The set of all MHP  $P^0$ between all node pairs is denoted by  $\mathcal{P}^0$ .

Although each satellite has four ISLs, any satellite on  $P_{AB}^0$  has at most two forwarding directions [6]. The next-hop node on  $P_{AB}^0$  is always closer to B in terms of hop-count. The required minimum hop-count of  $P_{AB}^0$  can be given by  $N(P_{AB}^0) = N^v(P_{AB}^0) + N^h(P_{AB}^0)$ , where  $N^v(P_{AB}^0) = \min\{|v_A - v_B|, M_P - |v_A - v_B|\}$  and  $N^h(P_{AB}^0) = \min\{|h_A - h_B|, N_P - |h_A - h_B|\}$  [1].

**Definition 2.** Horizontal and Vertical Detour. A nonminimum hop path contains one or multiple detours.  $P_{CD}$ is a horizontal detour if  $h_C = h_D$  and  $N^h(P_{CD}) > 0$ , as shown in Fig 3. Similarly,  $P_{CD}$  is a vertical detour if  $v_C = v_D$  and  $N^v(P_{CD}) > 0$ , as shown in Fig 4. Due to the mesh-like topology, a detour  $P_{CD}$  has more hops than  $P_{CD}^0$ .  $N(P_{CD}) - N(P_{CD}^0) = 2k$ ,  $k \in \mathbb{Z}^+$ .

**Definition 3.** Minimal Vertical Detour (MVD). If  $P_{CD}$  is a



Fig. 5. Minimal vertical detour(MVD) and single-hop vertical detour(SHVD).

vertical detour and all horizontal hops of  $P_{CD}$  are successively taken, then  $P_{CD}$  is defined as an MVD (see Fig. 5) and  $\mathcal{P}^{\text{MVD}}$ denotes its set . Note that each vertical detour contains at least one MVD. If  $P_{CD} \in \mathcal{P}^{\text{MVD}}$ , then  $N^v(P_{CD}) = 2k, k \in \mathbb{Z}^+$ .

**Definition 4.** Single-Hop Vertical Detour (SHVD). If  $P_{CD} \in \mathcal{P}^{\text{MVD}}$  and  $N^v(P_{CD}) = 2$ , then  $P_{CD}$  is defined as an SHVD (see Fig. 5).  $\mathcal{P}^{\text{SHVD}}$  denotes the SHVD set,  $\mathcal{P}^{\text{SHVD}} \subset \mathcal{P}^{\text{MVD}}$ .

**Definition 5.** Shortest Distance Path (SDP). The SDP between A and B is defined as  $P_{AB}^* = \arg \min_{P \in \{P_{AB}\}} d(P)$ . Let  $P^*$  denote an SDP between any given node pair.

#### IV. EQUIVALENT TRANSFORMATION OF PROPOSITIONS

The core proposition that all the SDPs belong to the MHP set in an LEO-SCN can be expressed as  $\mathbf{P1}$ :  $\forall P^* \in \mathcal{P}^0$ . This paper aims at judging the conditions for the establishment of **P1**. Based on the above definitions, we derive several simplified necessary and sufficient conditions for **P1**, and propose a discriminant function to judge if **P1** holds or not.

# Proposition 1. An SDP has no horizontal detour.

*Proof.* If  $P_{CD}$  is a horizontal detour, as shown in Fig. 3, then  $h_C = h_D$ ,  $N^h(P_{CD}) > 0$ , while  $N^h(P_{CD}^0) = 0$ . Since all the V-ISLs have the same physical distance  $d^V$ ,  $d(P_{CD}) = N^v(P_{CD}) \cdot d^V + \sum_{i=1}^{N^h(P_{CD})} d_i^H$ , where  $d_i^H > 0$  is the distance of *i*-th H-ISL; while  $d(P_{CD}^0) = N^v(P_{CD}^0) \cdot d^V$ . Since  $N^v(P_{CD}) \ge N^v(P_{CD}^0)$ , then  $d(P_{CD}) > d(P_{CD}^0)$ , the horizontal detour must be longer than the corresponding MHP. Because any segment of the SDP should also be an SDP, the horizontal detour cannot be included in an SDP.  $\Box$ 

Note that the distances of H-ISLs are different and vary with the satellite location. Although vertical detour introduces extra vertical hops, if vertical detour reaches the location with shorter H-ISLs and the saved distance of H-ISLs can make up for the extra vertical hops, then the detour is shorter than the MHP. Therefore, the vertical detour is possible in an SDP.

**Proposition 2.** A necessary and sufficient condition for  $\forall P^* \in \mathcal{P}^0$  is that  $\forall P_{CD} \in \mathcal{P}^{\text{MVD}}, d(P_{CD}) > d(P_{CD}^0)$ .

Proof. (1) Necessity: If  $P_{CD} \in \mathcal{P}^{\text{MVD}}$ , then  $v_C = v_D$  and  $P_{CD}^0$  is unique. Given  $\forall P^* \in \mathcal{P}^0$ , thus  $P_{CD}^* = P_{CD}^0$ . Since  $\mathcal{P}^{\text{MVD}} \cap \mathcal{P}^0 = \emptyset$ , if  $P_{CD} \in \mathcal{P}^{\text{MVD}}$ , then  $P_{CD} \notin \mathcal{P}^0$  and  $P_{CD} \neq P_{CD}^0 = P_{CD}^*$ . Thus  $d(P_{CD}) > d(P_{CD}^*) = d(P_{CD}^0)$ .

(2) Sufficiency: If  $\exists P^* \notin \mathcal{P}^0$ , then  $P^*$  has at least one detour. According to *Proposition 1*, the detour must be a vertical detour. Then there must be an MVD on the detour, and we take it as  $P_{CD}$ . Since any segment of  $P^*$  is also the shortest,



Fig. 6. H-ISL distance variation pattern.

then  $P_{CD} = P_{CD}^*$ ,  $d(P_{CD}) \leq d(P_{CD}^0)$ . That is, if  $\exists P^* \notin \mathcal{P}^0$ , then  $\exists P_{CD} \in \mathcal{P}^{\text{MVD}}$ ,  $d(P_{CD}) \leq d(P_{CD}^0)$ . The contrapositive proves the sufficiency of the original proposition.

**Proposition 3.** A necessary and sufficient condition for  $\forall P_{CD} \in \mathcal{P}^{\text{MVD}}$ ,  $d(P_{CD}) > d(P_{CD}^0)$  is that  $\forall P_{CD} \in \mathcal{P}^{\text{SHVD}}$ ,  $d(P_{CD}) > d(P_{CD}^0)$ .

*Proof.* (1) Necessity: Since  $\mathcal{P}^{SHVD} \subseteq \mathcal{P}^{MVD}$ , the necessity is obvious.

(2) Sufficiency: As shown in Fig. 5, given  $P_{CD} \in \mathcal{P}^{\text{MVD}}$ , then  $N^v(P_{CD}) = 2k, k \in \mathbb{Z}^+$ , and  $d(P_{CD}) = d(P_{C_kD_k}^0) + 2k \cdot d^V$ . If  $\forall P_{CD} \in \mathcal{P}^{\text{SHVD}}$ ,  $d(P_{CD}) > d(P_{CD}^0)$  is given, since  $(P_{CC_1} + P_{C_1D_1}^0 + P_{D_1D}) \in \mathcal{P}^{\text{SHVD}}$ , then  $d(P_{C_1D_1}^0) + 2d^V > d(P_{CD}^0)$ . Similarly,  $d(P_{C_2D_2}^0) + 2d^V > d(P_{C_1D_1}^0)$ , ...,  $d(P_{C_kD_k}^0) + 2d^V > d(P_{C_{k-1}D_{k-1}}^0)$ . Then  $d(P_{CD}) = d(P_{C_kD_k}^0) + 2k \cdot d^V > d(P_{CD}^0)$ . The sufficiency is proved.  $\Box$ 

Combining *Proposition* 2 and 3, to judge if  $\forall P^* \in \mathcal{P}^0$  is simplified to judge if  $\forall P_{CD} \in \mathcal{P}^{\text{SHVD}}$ ,  $d(P_{CD}) > d(P_{CD}^0)$ . Note that the above propositions apply to all LEO-SCNs with the four-ISL pattern and mesh-like topology. Although H-ISL distances vary with time (or satellite phase u), these propositions apply to cases at all times.

As shown in Fig. 5, if  $P_{CD} \in \mathcal{P}^{\text{SHVD}}$ ,  $d(P_{CD}) = d(P_{C_1D_1}^0) + 2d^V$ . The H-ISL distance varies with u and can be expressed as  $d^H(u)$ . Let the satellite phase of C be  $u_1$ , then u of the *i*-th node along  $P_{CD}^0$  is  $u_i = u_1 + (i-1)\Delta f$ , and u of the corresponding *i*-th node along  $P_{C_1D_1}^0$  is  $u_i \pm \Delta \Phi$ . Due to the symmetry of  $d^H(u)$ , analysis of solely  $u_i + \Delta \Phi$  will suffice. Let  $g(u_i) \triangleq d^H(u_i) - d^H(u_i + \Delta \Phi)$ . Assume  $N^h(P_{CD}^0) = m$ , then the distance difference between  $P_{CD}^0$  and  $P_{CD}$  is  $G(u_1,m) \triangleq d(P_{CD}^0) - d(P_{CD}) = \sum_{i=1}^m g(u_i) - 2d^V$ , where  $u_i = u_1 + (i-1)\Delta f$  and  $m = \{1, 2, ..., \lfloor \frac{N_P}{2} \rfloor\}$ .

**Proposition 4.** A necessary and sufficient condition for  $\forall P_{CD} \in \mathcal{P}^{\text{SHVD}}$ ,  $d(P_{CD}) > d(P_{CD}^0)$  is that  $\forall u_1 \in [\frac{-\Delta f - \Delta \Phi}{2}, \frac{\pi - \Delta f - \Delta \Phi}{2}]$ , we have the Discriminant Function  $G_M(u_1)$ , and  $G_M(u_1)$  should satisfy

$$G_{M}(u_{1}) \triangleq \sum_{i=1}^{M} d^{H}(u_{i}) - d^{H}(u_{i} + \Delta \Phi) - 2d^{V} < 0 \quad (4)$$
  
where  $M = \min\{\lfloor \frac{N_{P}}{2} \rfloor, \lfloor \frac{\pi - \Delta \Phi - 2u_{1}}{2\Delta f} + \frac{1}{2} \rfloor\}$  and  $u_{i} = u_{1} + (i-1)\Delta f$ .

*Proof.* If  $\forall P_{CD} \in \mathcal{P}^{\text{SHVD}}$ ,  $d(P_{CD}) > d(P_{CD}^0)$ , then  $\forall u_1, m$ ,  $G(u_1, m) < 0$ . The above two propositions are equivalent.

3

 TABLE I

 Relationship of SDP and MHP in typical constellations

Constellation	Parameter	$\forall P^* \in \mathcal{P}^0$
GlobalStar	52°: 48 / 8 / 0	Yes
Iridium-a	86.4°: 66 / 6 / 2	Yes
Iridium-b	86.4°: 66 / 6 / 0	No
OneWeb	87.9°: 648 / 18 / 0	No
Starlink-I	53°: 1584 / 72 / 39	Yes

Next, let  $G^{\max}$  be the maximum of  $G(u_1, m)$ , we search for  $G^{\max}$  and further narrow the range of  $u_1$  and m.

Given  $u_1$ ,  $G(u_1, i) - G(u_1, i-1) = g(u_i)$ . Let  $g(u_i) \ge 0$ , i.e.,  $d^H(u_i) \ge d^H(u_i + \Delta \Phi)$ , based on the monotonicity and symmetry of  $d^H(u)$  as discussed in Section II-B and shown in Fig. 6,  $u_i$  should satisfy  $-\frac{\Delta f}{2} \le \frac{u_i + (u_i + \Delta \Phi)}{2} \le \frac{\pi}{2} - \frac{\Delta f}{2}$ . Let  $u_i$  be  $u_1$ , we obtain  $\frac{-\Delta f - \Delta \Phi}{2\Delta f} \le u_1 \le \frac{\pi - \Delta f - \Delta \Phi}{2}$ ; let  $u_i$ be  $u_m$ , we obtain  $m \le \frac{\pi - \Delta \Phi^2 - 2u_1}{2\Delta f} + \frac{1}{2}$ . Besides, in the meshlike topology,  $m \le \lfloor \frac{N_P}{2} \rfloor$  [1]. Thus to keep  $g(u_i) \ge 0$ , the maximum of m is  $M = \min\{\lfloor \frac{N_P}{2} \rfloor, \lfloor \frac{\pi - \Delta \Phi - 2u_1}{2\Delta f} + \frac{1}{2} \rfloor\}$ . Then  $G(u_1, m)$  reaches  $G^{\max}$  at m = M, and  $G(u_1, M)$  is taken as the discriminant function and denoted by  $G_M(u_1)$ . The above constraints of  $u_1$  and m also specify the conditions when  $G(u_1, m)$  reaches  $G^{\max}$ . Meanwhile, if  $G_M(u_1) < 0$ with  $u_1$  in the specified range, then  $G^{\max} < 0$  and  $\forall u_1, m$ ,  $G(u_1, m) < 0$ . Thus, the ranges of  $u_1$  and m are narrowed.  $\Box$ 

 $G_M(u_1)$  actually means the maximum distance gap between the SHVD and MHP at  $u_1$ . Note that since near-polar constellations (e.g.,  $\alpha \in [80, 100] \text{ deg}$ ) are  $\pi$ -type [7], related values should be modified by  $\Delta \Omega = \pi/N_P$  and  $M = \min\{\lfloor N_P - 1 \rfloor, \lfloor \frac{\pi - \Delta \Phi - 2u_1}{2\Delta f} + \frac{1}{2} \rfloor\}$  in near-polar constellation cases.

Proposition 4 offers an analytical criterion to judge if P1 holds or not. Based on (1) and (4), whether P1 holds depends on four constellation parameter  $\alpha$ ,  $N_P$ ,  $M_P$ , and F, but not  $h_s$ .

In addition to the theoretical value, P1 helps to simplify the calculation of SDP. In a graph with n nodes, to solve the SDP between two nodes, traditional methods need to traverse all the n nodes. But if P1 holds, the SDP calculation only needs to search the MHP region. Averagely, only 1/16 computations are needed [7], saving over 93% computations.

#### V. SIMULATIONS AND RESULTS

Based on *Proposition 4* and the *discriminant function*, we examine if **P1** holds or not in some typical constellations, as listed in Table I. Based on Table I, we also test the *Proposition 4* by Monte Carlo simulations which are independent of the above formulations. For each constellation in which **P1** holds in Table I, we randomly generate  $1 \times 10^6$  node pairs at different time and calculate the shortest path and its hop-count between each pair, then check if it equals the minimum hop-count. The results support the conclusion that in these constellations all SDPs belong to the MHP set. The establishment of **P1** also means that the calculation of SDP in these constellations can be simplified and only the MHP sub-region is needed.

We further study the effects of the four factors  $\alpha$ ,  $N_P$ ,  $M_P$ , and F by examining if **P1** holds with all the possible parame-



4

Fig. 7. The maximum of the discriminant function with different  $\alpha$  and F (at fixed  $N_P = 18$ ,  $M_p = 36$ ).  $G_M^{max} = 0$  is the critical plane.  $G_M^{max}$  below the plane means **P1** holds in the corresponding constellation.

ter combinations.  $N_P, M_P = [6, 7, ..., 100], F = [0, 1, ..., N_P - 1], \alpha \in (0, 120]$  deg and is discretized by 0.5 deg. Results show that **P1** always holds for any inclined constellations with  $\alpha \leq 68$  deg. When  $\alpha > 68$  deg, the establishment of **P1** is mainly affected by  $\alpha$  and F rather than  $N_P$  or  $M_P$ .

Fig. 7 gives an example of  $N_P = 18$ ,  $M_p = 36$ . **P1** holds in most cases, but the surface above the critical plane indicates that **P1** does not hold when  $\alpha$  approaches 90 deg and F is small. Based on  $d^H(u)$ , when  $\alpha$  approaches 90 deg,  $\left|\frac{dd}{du}\right|$  and  $g(u_i)$  are greater, then  $G_M^{max}$  is greater; Similarly, a smaller F allows greater M, which also leads to a greater  $G_M^{max}$ . In these cases, **P1** does not hold, i.e., it is possible to find a path with vertical detour that has a shorter distance than the MHP.

#### VI. CONCLUSION

This paper derives some simplified equivalent propositions when all SDPs belong to the MHP set and provides a discriminant function to judge if the proposition holds or not. Theoretical model and simulations show that the proposition holds in constellations with small inclinations (e.g., less than 68 deg) or large phasing offsets.

#### REFERENCES

- Q. Chen *et al.*, "Multiple gateway placement in large-scale constellation networks with inter-satellite links," *Int. J. Satell. Commun. Netw.*, vol. 39, no. 1, pp. 47–64, 2021.
- [2] T. de Cola *et al.*, "Network and protocol architectures for future satellite systems," *Found. Trends Netw.*, vol. 12, no. 1-2, pp. 1–161, 2017.
- [3] M. Handley, "Delay is not an option," HotNet 2018, pp. 85-91, 2018.
- [4] M. Mohorcic, A. Svigelj, and G. Kandus, "Traffic class dependent routing in ISL networks," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 40, no. 4, pp. 1160–1172, 2004.
- [5] Z. Qu et al., "LEO satellite constellation for internet of things," IEEE Access, vol. 5, pp. 18391–18401, 2017.
- [6] E. Ekici, I. F. Akyildiz, and M. D. Bender, "A distributed routing algorithm for datagram traffic in LEO satellite networks," *IEEE ACM Trans. Netw.*, vol. 9, no. 2, pp. 137–147, 2001.
- [7] Q. Chen *et al.*, "A distributed congestion avoidance routing algorithm in mega-constellation network with multi-gateway," *Acta Astronautica*, vol. 162, pp. 376–387, 2019.
- [8] S. Duan, "Research on distributed routing algorithm in satellite networks based on LEO polar orbits constellation," Thesis, 2014.
- [9] M. Besta *et al.*, "High-performance routing with multipathing and path diversity in ethernet and HPC networks," *IEEE Trans. Parallel Distrib. Syst.*, vol. 32, no. 4, pp. 943–959, 2021.



Quan Chen Quan Chen received the B.E. and Ph.D. degrees in 2015 and 2021, respectively, from the National University of Defense Technology (NUDT), Changsha, China. He is currently a lecturer with the College of Aerospace Science and Engineering, NUDT. From 2019 to 2020, he was a Visiting Scholar with the Department of Electrical Engineering, KU Leuven, Belgium. His research interests include mega-constellation satellite networks, UAV networks, and integrated space-terrestrial networks.



Xiaoqian Chen received his M.S. and Ph.D. degrees in aerospace engineering from the National University of Defense Technology, China, in 1997 and 2001, respectively. He is currently a professor and the dean of the National Institute of Defense Technology Innovation, Beijing, China. His current research interests include spacecraft systems engineering, advanced digital design methods of space systems, and multidisciplinary design optimization.

5



Lei Yang received his Ph.D. degree from the College of Aerospace Science and Engineering, National University of Defense Technology, Changsha, China in 2008. Prof. Yang is currently a member of the Chinese Society of Astronautics and China Instrument and Control Society. His current research interests are focused on satellite communication networks, measurement and control technology for microsatellite, on-board computer, spacecraft system modeling, and simulation.



**Deke Guo** received the B.S. degree in industry engineering from the Beijing University of Aeronautics and Astronautics, Beijing, China, in 2001, and the Ph.D. degree in management science and engineering from the National University of Defense Technology, Changsha, China, in 2008. He is currently a Professor with the College of System Engineering, National University of Defense Technology, and also with the College of Intelligence and Computing, Tianjin University. His research interests include distributed systems, software-defined networking, data

center networking, wireless and mobile systems, and interconnection networks. He is a member of the ACM.



**Bangbang Ren** received the B.S. degree and Ph.D. degree in management science and engineering from National University of Defense Technology in 2015 and 2021, respectively. His research interests include software defined network, network function virtualization, approximation algorithm.



Jianming Guo received the B.S. degree from the University of Science and Technology of China, Hefei, China in 2015, and the Ph.D. degree from the National University of Defense Technology, Changsha, China in 2021. He was a visiting PhD student at Universitat Politècnica de Catalunya (UPC) from 2019 to 2020. His current research interests include SDN, NFV, and satellite communication networks.